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ICS 312
Homework #1
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Exercise #1: Conversions [5pts]

Perform the following conversions, and show the details of the work

a. Hexadecimal $4AD3_h$ into binary

Answer:

4	A	D	3
↓	↓	↓	↓
0100	1010	1101	0011

So, the binary expression of $4AD3_h$ is $0100\ 1010\ 1101\ 0011_b$.

b. Binary $1\ 0111\ 1010\ 1011_b$ into hexadecimal

Answer:

0001	0111	1010	1011
↓	↓	↓	↓
1	7	A	B

So, the hexadecimal expression of $1\ 0111\ 1010\ 1011_b$ is $17AB_h$.

c. Decimal 231_d into binary

Answer:

231 / 2 = 115	+ 1
115 / 2 = 57	+ 1
57 / 2 = 28	+ 1
28 / 2 = 14	+ 0
14 / 2 = 7	+ 0
7 / 2 = 3	+ 1
3 / 2 = 1	+ 1
1 / 2 = 0	+ 1

So, the hexadecimal expression of 231_d is $1110\ 0111_b$.

d. Decimal 677_d into hexadecimal

Answer:

677 / 16	= 42	+ 5
42 / 16	= 2	+ 10
2 / 16	= 0	+ 2

So, the hexadecimal expression of 677_d is $2A5_h$.

e. Hexadecimal $A32_h$ into decimal

Answer:

$$10 \times 16^2 + 3 \times 16^1 + 2 \times 16^0$$
$$= 2560 + 48 + 2$$
$$= 2610$$

So, the hexadecimal expression of $A32_h$ is 2610_d.

Exercise #2: Two's Complementation [15pts]

a. Give the binary 16-bit two's complement representation of the following decimal integers, and show the details of the work

1. 341
2. -1
3. 0
4. -56
5. -101

Answer:

Convert 341_d:

341 / 2	= 170	+ 1
170 / 2	= 85	+ 0
85 / 2	= 42	+ 1
42 / 2	= 21	+ 0
21 / 2	= 10	+ 1

10 / 2	= 5	+ 0
5 / 2	= 2	+ 1
2 / 2	= 1	+ 0
1 / 2	= 0	+ 1

So, the 16-bit two's complement representation of 341_d is 0000 0001 0101 0101_b.

Convert -1_d:

1 / 2	= 0	+ 1

Flip:

0000 0000 0000 0001 ⇒ 1111 1111 1111 1110

Plus one:

1111 1111 1111 1110 ⇒ 1111 1111 1111 1111

So, the 16-bit two's complement representation of -1_d is 1111 1111 1111 1111_b.

Flip 0_d:

0000 0000 0000 0000 ⇒ 1111 1111 1111 1111

Plus one:

1111 1111 1111 1111 ⇒ 0000 0000 0000 0000

So, the 16-bit two's complement representation of 0_d is 0000 0000 0000 0000_b.

Convert -56_d:

56 / 2	= 28	+ 0
28 / 2	= 14	+ 0
14 / 2	= 7	+ 0
7 / 2	= 3	+ 1
3 / 2	= 1	+ 1
1 / 2	= 0	+ 1

Flip:

0000 0000 0011 1000 ⇒ 1111 1111 1100 0111

Plus one:

1111 1111 1100 0111 ⇒ 1111 1111 1100 1000

So, the 16-bit two's complement representation of -56_d is 1111 1111 1100 1000_b.

Convert -101$_d$:

101 / 2	= 50	+ 1
50 / 2	= 25	+ 0
25 / 2	= 12	+ 1
12 / 2	= 6	+ 0
6 / 2	= 3	+ 0
3 / 2	= 1	+ 1
1 / 2	= 0	+ 1

Flip:

0000 0000 0110 0101 ⇒ 1111 1111 1001 1010

Plus one:

1111 1111 1001 1010 ⇒ 1111 1111 1001 1011

So, the 16-bit two's complement representation of -101$_d$ is 1111 1111 1001 1011$_b$.

b. Give the hexadecimal 32-bit two's complement representation of the following decimal integers, and show the details of the work

1. 783
2. -1
3. -1,321
4. -32
5. 421

Answer:

Convert 783$_d$:

783 / 16	= 48	+ 15
48 / 16	= 3	+ 0
3 / 16	= 0	+ 3

So, the hexadecimal 32-bit two's complement representation of 783$_d$ is 30F$_h$.

Convert -1$_d$:

1 / 16	= 0	+ 1

Flip:

0000 0001 ⇒ FFFF FFFE

Plus one:

FFFF FFFE ⇒ FFFF FFFF

So, the hexadecimal 32-bit two's complement representation of -1_d is FFFF FFFF $_h$.

Convert -1321_d:

1,321 / 16	= 82	+ 9
82 / 16	= 5	+ 2
5 / 16	= 0	+ 5

Flip:

0000 0529	⇒	FFFF FAD6

Plus one:

FFFF FAD6	⇒	FFFF FAD7

So, the hexadecimal 32-bit two's complement representation of $-1{,}321_d$ is FFFF FFAD7 $_h$.

Convert -32_d:

32 / 16	= 2	+ 0
2 / 16	= 0	+ 2

Flip:

0000 0020	⇒	FFFF FFDF

Plus one:

FFFF FFDF	⇒	FFFF FFE0

So, the hexadecimal 32-bit two's complement representation of -32_d is FFFF FFE0$_h$.

Convert 421_d:

421 / 16	= 26	+ 5
26 / 16	= 1	+ 10
1 / 16	= 0	+ 1

So, the hexadecimal 32-bit two's complement representation of 421_d is 1A5$_h$.

c. Give the decimal value of the following two's complement hexadecimal representations and show the details of the work

1. A34
2. 81F
3. D73
4. 7AA
5. 63A

Answer:

Flip $A34_h$:

A34 ⇒ 5CB

Plus one:

5CB ⇒ 5CC

Convert:

$$-(5 \times 16^2 + 12 \times 16^1 + 12 \times 16^0)$$
$$= -(1280 + 192 + 12)$$
$$= -1484$$

So, the decimal value of the two's complement representation $A34_h$ is -1484_d.

Flip $81F_h$:

81F ⇒ 7E0

Plus one:

7E0 ⇒ 7E1

Convert:

$$-(7 \times 16^2 + 14 \times 16^1 + 1 \times 16^0)$$
$$= -(1792 + 224 + 1)$$
$$= -2017$$

So, the decimal value of the two's complement representation $81F_h$ is -2017_d.

Flip $D73_h$:

D73 ⇒ 28C

Plus one:

28C ⇒ 28D

Convert:

$$-(2 \times 16^2 + 8 \times 16^1 + 13 \times 16^0)$$
$$= -(512 + 128 + 13)$$
$$= -653$$

So, the decimal value of the two's complement representation $D73_h$ is -653_d.

Convert $7AA_h$:

$$7 \times 16^2 + 10 \times 16^1 + 10 \times 16^0$$
$$= 1792 + 160 + 16$$
$$= 1968$$

So, the decimal value of the two's complement representation $7AA_h$ is 1968_d.

Convert $63A_h$:

$$6 \times 16^2 + 3 \times 16^1 + 10 \times 16^0$$
$$= 1536 + 48 + 10$$
$$= 1594$$

So, the decimal value of the two's complement representation $63A_h$ is 1594_d.

Exercise #3: Additions and Subtractions [8pts]

a. Perform the following binary arithmetic operations, and show carries.

1. 1010111 + 101101
2. 1000010 + 0111011
3. 1101110 + 1111011
4. 0001111 + 1010101

Answer:

```
          c1c 0c 1c 0c 1c 1c 1
+              1  0  1  1  0  1
-------------------------------------
          1  0  0  0  0  1  0  0
```

Therefore, $101\ 0111_b + 10\ 1101_b = 1000\ 0100_b$

1 0 0 0 0^{c}1 0
\+ 0 1 1 1 0 1 1
\-------------------------------------
1 1 1 1 1 0 1

Therefore, 100 0010$_b$ + 011 1011$_b$ = 111 1101$_b$

^{c}1^{c}1^{c}0^{c}1^{c}1^{c}1 0
\+ 1 1 1 1 0 1 1
\-------------------------------------
1 1 1 0 1 0 0 1

Therefore, 110 1110$_b$ + 111 1011$_b$ = 1110 1001$_b$

0 0^{c}0^{c}1^{c}1^{c}1^{c}1
\+ 1 0 1 0 1 0 1
\-------------------------------------
1 1 0 0 1 0 0

Therefore, 0001111$_b$ + 1010101$_b$ = 1100100$_b$

b. Perform the following binary arithmetic operations, and show carries.

1. A3F1E + 32AA
2. FF31A + BB4FB
3. 13111 + FFFFF
4. 32AAA + F0F0F

Answer:

A 3^{c}F 1^{c}E
\+ 3 2 B A A
\-----------------------------
D 6 A C 8

Therefore, A3F1E$_h$ + 32BAA$_h$ = D6AC8$_h$

^{c}F^{c}F 3^{c}1^{c}A
\+ B B 4 F B
\-----------------------------
1 B A 8 1 5

Therefore, FF31A$_h$ + BB4FB$_h$ = 1BA815$_h$

$$\begin{array}{rcccccc} & & {}^{c}1 & {}^{c}3 & {}^{c}1 & {}^{c}1 & {}^{c}1 \\ + & & F & F & F & F & F \\ \hline & 1 & 1 & 3 & 1 & 1 & 0 \end{array}$$

Therefore, 13111_h + $FFFFF_h$ = 113110_h

$$\begin{array}{rcccccc} & & {}^{c}3 & 2 & {}^{c}A & A & {}^{c}A \\ + & & F & 0 & F & 0 & F \\ \hline & 1 & 2 & 3 & 9 & B & 9 \end{array}$$

Therefore, $32AAA_h$ + $F0F0F_h$ = $1239B9_h$