TA: Jade Cheng

ICS 241

Recitation Lecture Note #5

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Question: Let R be the following relation defined on the set $\{a, b, c, d\}$:

$$R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$$

Determine whether R is [Chapter 8.1 Review]

a. Reflexive

Answer: R is reflexive because R contains (a, a), (b, b), (c, c), and (d, d).

b. symmetric

Answer: R is not symmetric because $(a, c) \in R$, but $(c, a) \notin R$.

c. antisymmetric

Answer: R is not antisymmetric because $(b, c) \in R$, and $(c, b) \in R$,.but $b \neq c$.

d. transitive

Answer: R is not transitive because, for example, $(a, c) \in R$ and $(c, b) \in R$, but $(a, b) \notin R$.

Question: Let R be the following relation on the set of real numbers:

$$aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor, \text{where } \lfloor x \rfloor \text{ is the floor of } x.$$

Determine whether R is [Chapter 8.1 Review]

a. Reflexive

Answer: R is reflexive because $\lfloor a \rfloor = \lfloor a \rfloor$ is true from all real numbers.

b. symmetric

Answer: R is symmetric suppose $\lfloor a \rfloor = \lfloor b \rfloor$; then $\lfloor b \rfloor = \lfloor a \rfloor$.

c. antisymmetric

Answer: R is not antisymmetric: we can have aRb and bRa for distinct a and b. For example, $\lfloor 1.1 \rfloor = \lfloor 1.2 \rfloor$.

d. transitive

Answer: R is transitive, suppose $\lfloor a \rfloor = \lfloor b \rfloor$ and $\lfloor b \rfloor = \lfloor c \rfloor$; from transitivity of equality of real numbers, it follows that $\lfloor a \rfloor = \lfloor c \rfloor$.

Question: Answer the following questions regarding this Flight Table: [Chapter 8.2 Review]

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
N	122	34	Detroit	08:10
A	221	22	Denver	08:17
A	122	33	Anchorage	08:22
A	323	34	Honolulu	08:30
N	199	13	Detroit	08:47
A	222	22	Denver	09:10
N	322	34	Detroit	09:44

a. Assuming that no new $\boldsymbol{n}$-tuples are added, find a composite key with two fields containing the *Airline* field for the database in the Flight Table.

Answer: *Airline* and *Flight_number*; *Airline* and *Departure_time*

b. What do you obtain when you apply the selection operator $\boldsymbol{s_C}$, where $\boldsymbol{C}$ is the condition *Destination* = Detroit, to the data base in the Flight Table.

Answer: (N, 122, 34, Detroit, 08:10), (N, 199, 13, Detroit, 08:47), (N, 322, 34, Detroit, 09:44).

c. What do you obtain when you apply the selection operator $\boldsymbol{s_C}$, where $\boldsymbol{C}$ is the condition (*Airline* = N) ∨ (*Destination* = Denver), to the data base in the Flight Table.

Answer: (N, 122, 34, Detroit, 08:10), (N, 199, 13, Detroit, 08:47), (N, 322, 34, Detroit, 09:44), (A, 221, 22, Denver, 08:17), (A, 222, 22, Denver, 09:10).

d. Display the table produced by applying the projection $\boldsymbol{P_{1,4}}$ to the Flight Table.

Answer:

Airline	*Destination*
N	Detroit
A	Denver
A	Anchorage
A	Honolulu
N	Detroit
A	Denver
N	Detroit

Question: Show that if C_1 and C_2 are conditions that elements of the n-ary relation R may satisfy, then $s_{C_1}(s_{C_2}R) = s_{C_2}(s_{C_1}R)$. [Chapter 8.2 Review]

Answer: Both sides of this equation pick out the subset of R consisting of those n-tuples satisfying both conditions C_1 and C_2.

$$s_{C_1}(s_{C_2}R) = s_{C_1,C_2}R$$

$$s_{C_2}(s_{C_1}R) = s_{C_1,C_2}R$$

$$\Rightarrow s_{C_1}(s_{C_2}R) = s_{C_2}(s_{C_1}R).$$

Question: Give an example to show that if R and S are both n-ary relations, then $P_{i_1,i_2,\cdots i_m}(R-S)$ may be different from $P_{i_1,i_2,\cdots i_m}(R) - P_{i_1,i_2,\cdots i_m}(S)$. [Chapter 8.2 Review]

Answer: Let set R be the n-tuple relation below:

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
N	122	34	Detroit	08:10
A	221	22	Denver	08:17
A	122	33	Anchorage	08:22
A	323	34	Honolulu	08:30

Let set S be the n-tuple relation below:

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
A	221	22	Denver	08:17
A	122	33	Anchorage	08:22
A	323	34	Honolulu	08:30
N	199	13	Detroit	08:47
A	222	22	Denver	09:10
N	322	34	Detroit	09:44

By definition $R - S$ contains the set elements that are in R but not in S. So $R - S$ is as below:

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
N	122	34	Detroit	08:10

Let $m = 1$, We obtain the following relationship regarding $P_{i_1,i_2,\cdots i_m}(R - S)$ and $P_{i_1,i_2,\cdots i_m}(R) - P_{i_1,i_2,\cdots i_m}(S)$:

$$P_{i_1,i_2,\cdots i_m}(R - S) = P_1(R - S)$$

$$= \{(\mathrm{N})\}\,.$$

$$P_{i_1,i_2,\cdots i_m}(R) - P_{i_1,i_2,\cdots i_m}(S) = P_1(R) - P_1(S)$$

$$P_1(R) = \{(\mathrm{N}), (\mathrm{A})\}$$

$$P_1(S) = \{(\mathrm{N}), (\mathrm{A})\}$$

$$\Rightarrow P_1(R) - P_1(S) = \emptyset\,.$$

$$\because \emptyset \neq \{(\mathrm{N})\}$$

$$\therefore P_{i_1,i_2,\cdots i_m}(R - S) \neq P_{i_1,i_2,\cdots i_m}(R) - P_{i_1,i_2,\cdots i_m}(S)\,.$$

Question: Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order) [Chapter 8.3 Review]

a. $\{(1, 1), (1, 2), (1, 3)\}$

Answer: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

b. $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

Answer: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

c. $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

Answer: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$

Question: List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). [Chapter 8.3 Review]

a. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Answer: $\{(1,1), (1,3), (2,2), (3,1), (3,3)\}$.

b. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Answer: $\{(1,2), (2,2), (3,2)\}$.

c. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Answer: $\{(1,1), (2,2), (1,3)\ (2,1)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$.

Question: Determine whether the relations represented by the matrices in the previous **Question** are reflexive, irreflexive, symmetric, antisymmetric. [Chapter 8.3 Review]

Answer: The matrix representing a reflexive relation has 1's for all its main diagonal entries such as:

$$\begin{bmatrix} 1 & ? & ? \\ ? & 1 & ? \\ ? & ? & 1 \end{bmatrix}.$$

The matrix representing a irreflexive relation has 0's for all its main diagonal entries such as

$$\begin{bmatrix} 0 & ? & ? \\ ? & 0 & ? \\ ? & ? & 0 \end{bmatrix}.$$

The matrix representing a symmetric relation has a symmetric 1's and 0's layout according to its main diagonal, such as

$$\begin{bmatrix} ? & ? & 0 \\ ? & ? & 1 \\ 0 & 1 & ? \end{bmatrix}.$$

The matrix representing an antisymmetric relation has all 1's having 0's as the counterpart according to its main diagonal, such as

$$\begin{bmatrix} ? & 1 & 0 \\ 0 & ? & 0 \\ 0 & 1 & ? \end{bmatrix}.$$

So it's clear $\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\1 & 0 & 1\end{bmatrix}$ is reflexive, and symmetric, $\begin{bmatrix}0 & 1 & 0\\0 & 1 & 0\\0 & 1 & 0\end{bmatrix}$ is antisymmetric, $\begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\1 & 1 & 1\end{bmatrix}$ is symmetric.

Question: Let R be the relation represented by the matrix: [Chapter 8.3 Review]

$$M_R = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix}$$

Find the matrices that represent R^2, R^3, and R^4.

Answer:

$$R^2 = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix}.\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\1 & 1 & 0\\0 & 1 & 1\end{bmatrix}.$$

$$R^3 = R^2.\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\1 & 1 & 0\\0 & 1 & 1\end{bmatrix}.\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 1 & 1\end{bmatrix}.$$

$$R^4 = R^3.\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 1 & 1\end{bmatrix}.\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}.$$

Some Problems that appeared in HW#2 and E1

Question: Find the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 17$, where $x_i, i = 1, 2, 3, 4$, are nonnegative integers such that $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5$, and $x_4 \leq 8$. [Chapter 7.6 Review]

Answer: To apply the principle of inclusion-exclusion, let a solution have property P_1 if $x_1 > 3$, property P_2 if $x_2 > 4$, property P_3 if $x_3 > 5$, and property P_4 if $x_4 > 8$. The number of solutions satisfying the inequalities $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5$ and $x_4 \leq 8$ is

$$N(P_1'P_2'P_3'P_4') = N - N(P_1) - N(P_3) - N(P_3) - N(P_4)$$

$$+N(P_1P_2) + N(P_1P_3) + N(P_1P_4) + N(P_2P_3) + N(P_2P_4) + N(P_3P_4)$$

$$-N(P_1P_2P_3) - N(P_1P_3P_4) - N(P_1P_2P_4) - N(P_2P_3P_4)$$

$$+N(P_1P_2P_3P_4)\ .$$

$N =$ total number of solutions $= C(4+17-1, 17) = 1140$
$N(P_1) =$ number of solutions with $x_1 > 3 = C(4+13-1, 13) = 560$
$N(P_2) =$ total number of solutions $x_2 > 4 = C(4+12-1, 12) = 455$
$N(P_3) =$ total number of solutions $x_3 > 5 = C(4+11-1, 11) = 364$
$N(P_4) =$ total number of solutions $x_4 > 8 = C(4+8-1, 8) = 165$
$N(P_1P_2) =$ total number of solutions $x_1 > 3$ and $x_2 > 4 = C(4+8-1, 8) = 165$
$N(P_1P_3) =$ total number of solutions $x_1 > 3$ and $x_3 > 5 = C(4+7-1, 7) = 120$
$N(P_1P_4) =$ total number of solutions $x_1 > 3$ and $x_4 > 8 = C(4+4-1, 4) = 35$
$N(P_2P_3) =$ total number of solutions $x_2 > 4$ and $x_3 > 5 = C(4+6-1, 6) = 84$
$N(P_2P_4) =$ total number of solutions $x_2 > 4$ and $x_4 > 8 = C(4+3-1, 3) = 20$
$N(P_3P_4) =$ total number of solutions $x_3 > 5$ and $x_4 > 8 = C(4+2-1, 2) = 10$
$N(P_1P_2P_3) =$ total number of solutions $x_1 > 3, x_2 > 4, x_3 > 5 = C(4+2-1, 2) = 10$
$N(P_1P_2P_4) =$ total number of solutions $x_1 > 3, x_2 > 4, x_4 > 8 = 0$
$N(P_1P_3P_4) =$ total number of solutions $x_1 > 3, x_3 > 5, x_4 > 8 = 0$
$N(P_2P_3P_4) =$ total number of solutions $x_2 > 4, x_3 > 5, x_4 > 8 = 0$
$N(P_1P_2P_3P_4) =$ total number of solutions $x_1 > 3, x_2 > 4, x_3 > 5, x_4 > 8 = 0$

Inserting these quantities into the formula for $N(P_1'P_2'P_3'P_4')$ shows that the number of solutions with that $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5,$ and $x_4 \leq 8$ equals: $N(P_1'P_2'P_3'P_4') = 1140 - 560 - 455 - 364 - 165 + 165 + 120 + 35 + 84 + 20 + 10 - 10 - 0 - 0 - 0 + 0 = 20$.

Question: Determine which of these are linear homogeneous recurrence relations with constant coefficients. Also, find the degree of those that are. [Chapter 7.2]

a. $a_n = 3a_{n-2}$

Answer: Linear, homogeneous, with constant coefficients, degree 2.

b. $a_n = 3$

Answer: Linear, with constant coefficients but not homogeneous.

c. $a_n = a_{n-1}^2$

Answer: not Linear.

d. $a_n = a_{n-1} + 2a_{n-3}$

Answer: Linear, homogeneous, with constant coefficients, degree 3.

e. $a_n = a_{n-1}/n$

Answer: Linear, homogeneous, but not with constant coefficients.

f. $a_n = a_{n-1} + a_{n-2} + n + 3$

Answer: Linear, with constant coefficients, but not homogeneous.

g. $a_n = 4a_{n-2} + 5a_{n-4} + 9a_{n-7}$

Answer: Linear, homogeneous, with constant coefficients, degree 7.

Question: Solve the recurrence relation $a_n = 2a_{n-1} + 2n^2$ with initial condition $a_1 = 4$. [Chapter 7.2 Review]

Answer: The characteristic equation for the associated homogeneous recurrence relation is $r - 2 = 0$. So it has solutions $r = 2$ Therefore the general solution to the associated homogeneous recurrence relation is:

$$a_n = a \cdot 2^n .$$

To obtain a particular solution to the given recurrence relation, try $a_n^{(p)} = b \cdot n^2 + c \cdot n + d$, obtaining:

$$b \cdot n^2 + c \cdot n + d = 2b \cdot (n-1)^2 + 2c \cdot (n-1) + 2d + 2n^2$$

$$\Longrightarrow (b+2)n^2 + (c-4b)n + 2b - 2c + d = 0$$

The coefficient of n^2, n terms and the constant term must each equal 0. Therefore, we have

$$b + 2 = 0$$
$$c - 4b = 0$$
$$2b - 2c + d = 0$$

Hence, we have $b = -2$ $c = -8$ and $d = -12$. Therefore:

$$a_n = a \cdot 2^n - 2n^2 - 8n - 12.$$

The initial condition $a_1 = 4$, yields the system of equations coefficient:

$$a_1 = a \cdot 2^1 - 2 \cdot 1^2 - 8 \cdot 1 - 12 = 4$$

$$\Longrightarrow a = 13\,.$$

The coefficient is found to be $a = 13$. Therefore the solution to the given recurrence relation is:

$$a_n = 13 \cdot 2^n - 2n^2 - 8n - 12.$$