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ICS 241

Recitation Lecture Notes #14

December 04, 2009

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Question: Find five other valid sentences, besides those given in Exercise 1 [Chapter 12.1 Review]

Answer: There are of course a large number of possible answers.

E.g. 1: sentence

=>	(noun phrase | intransitive verb phrase)			
=>	(article	adjective	noun)	intransitive verb phrase	
=>	article	adjective	noun	(intransitive verb | adverb)
=>	*The*	*sleepy*	*hare*	*runs*	*quickly*

E.g. 2: sentence

=>	(noun phrase	intransitive verb phrase)			
=>	(article	adjective	noun)	intransitive verb phrase	
=>	article	adjective	noun	(intransitive verb	adverb)
=>	*The*	*happy*	*hare*	*runs*	*slowly*

E.g. 3: sentence

=>	(noun phrase	transitive verb phrase)				
=>	(article	adjective	noun)	transitive verb phrase		
=>	article	adjective	noun	(transitive verb	noun phrase)	
=>	article	adjective	noun	transitive verb	(article	noun)
=>	*The*	*happy*	*tortoise*	*passes*	*the*	*hare*

E.g. 4: sentence

=>	(noun phrase	transitive verb phrase)			
=>	(article	noun)	transitive verb phrase		
=>	article	noun	(transitive verb	noun phrase)	
=>	article	noun	transitive verb	(article	noun)
=>	*The*	*hare*	*passes*	*the*	*tortoise*

E.g. 5: sentence

=>	(noun phrase	transitive verb phrase)		
=>	(article	noun)	transitive verb phrase	
=>	article	noun	(transitive verb	noun phrase)

=>	article	noun	transitive verb	(article	adjective	noun)
=>	*The*	*tortoise*	*passes*	*the*	*happy*	*hare*

Question: Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{0, 1, A, S\}$, $T = \{0, 1\}$, and set of productions P consisting of $S \to S1$, $S \to 0A00$, $A \to 0A$, and $A \to 11$. [Chapter 12.1 Review]

a. Show that 001100 belongs to the language generated by G.

Answer: It suffices to give a derivation of this string. We write the derivation in the obvious way.

$$S \to S1 \to 0A001 \to 00A001 \to 001100\,.$$

b. Show that 1010 does not belong to the language generated by G.

Answer: Every production results in a string that starts in S, or 0. S eventually starts with 0. The strings belonging to the language generated by G, therefore, have to start with 0. The given string, which starts with a 1 can not be generated.

c. What is the language generated by G?

Answer: Notice that we can have any number of 1's at the end of the string by iterating the production $S \to S1$. Eventually the S must turn into $0A00$, so at least three 0's must come before the one. We can then have as many 0's as we like by using the production $A \to 0A$ repeatedly. We must end up with at least one more 0 (and therefore a total of at least four 0's). In the middle of zeros, A disappears only upon using $A \to 11$. So the language generated by G is the set of all strings that can be expressed as $\{0^m 1100 1^n | m \geq 1 \text{ and } n \geq 0\}$.

Question: Let G be the grammar with $V = \{p, q, r, S\}$; $T = \{p, q, r\}$; starting symbol S; and productions $S \to pSr$, $S \to rqS$, $S \to rr$, and $S \to pqr$. Construct derivation trees for $rqppqrr$. [Chapter 12.1 Review]

Answer: If we look at the beginning of the string, we see that we can use the rule $S \to rqS$ first. Then since the remainder of the string (after the initial rq) starts with p, we can use the rule $S \to pSr$. Finally, we can use the rule $S \to pqr$. We therefore obtain the tree shown below.

```
      S
    / | \
   r  q  S
       / | \
      p  S  r
       / | \
      p  q  r
```

Question: Find a phrase-structure grammar for each of these languages [Chapter 12.1 Review]

a. the set consisting of the bit strings 10, or 0^n10, where $n > 0$

Answer: The set of bit strings is actually 0^n10, where $n \geq 0$. The grammar can be expressed as,

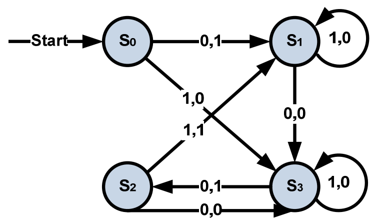
$$S \to 10$$
$$S \to 0S$$

b. the set of bit strings consisting of an even number of 0's following a leading final 1.

Answer: The set of bit string can be written as $1(00)^n$, where $n \geq 0$. The grammar can be expressed as

$$S \to 1A$$
$$A \to 00A$$
$$A \to \lambda$$

Question: Give the state table for the finite-state machines with the state diagram as shown below. [Chapter 12.2 Review]



Answer: Textbook's way:

State	f		g	
	Input		Input	
	0	1	0	1
S_0	S_1	S_3	1	0
S_1	S_3	S_1	0	0
S_2	S_3	S_1	0	1
S_3	S_2	S_3	1	0

Lecture notes' way:

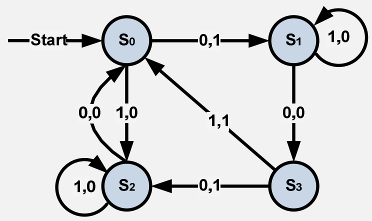
State	Input	
	0	1
S_0	$S_1, 1$	$S_3, 0$
S_1	$S_3, 0$	$S_1, 0$
S_2	$S_3, 0$	$S_1, 1$
S_3	$S_2, 1$	$S_3, 0$

Question: Solve the two problems based on the following state table for a finite-state machine. [Chapter 12.2 Review]

State	f		g	
	Input		Input	
	0	1	0	1
S_0	S_1	S_2	1	0
S_1	S_3	S_1	0	0
S_2	S_0	S_2	0	0
S_3	S_2	S_0	1	1

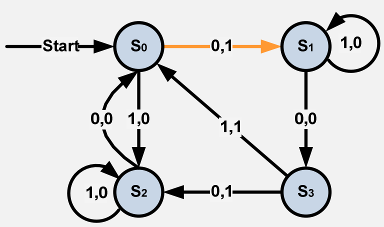
a. Find the state diagram to represent this finite-state machine.

Answer: The state diagram corresponds to the given state table is shown below.

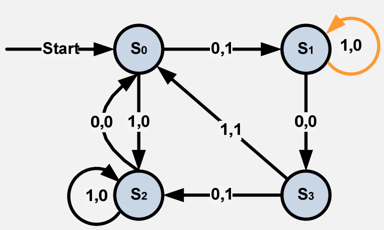


b. Find the output generated from the input string 0110 for the finite-state machine with the state diagram as below.

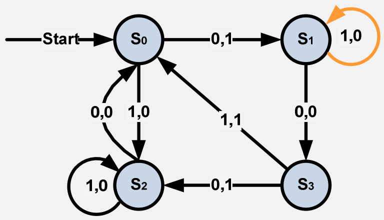
Answer: We follow the steps of each token of the input bit string gets consumed by the state machine.



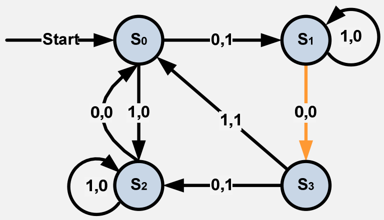
Step 1. Input 0, output string: 1



Step 2. Input 1, output string: 10



Step 3. Input 1, output string: 100



Step 4. Input 0, output string 1000

Question: Determine whether the string 10101000 is in each of these sets. [Chapter 12.3 Revierw]

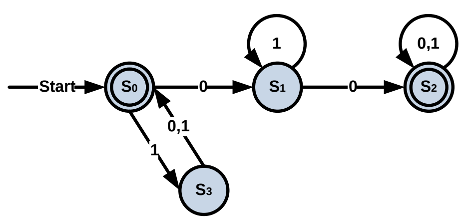
a. $\{101\}^*$

Answer: No. This set of strings need to end with a 1, but our string ends with an 0.

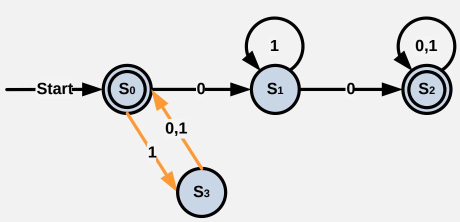
b. $\{01\}^*\{0\}^*$

Answer: Yes. Our string is $\{01\}^3\{0\}^2$.

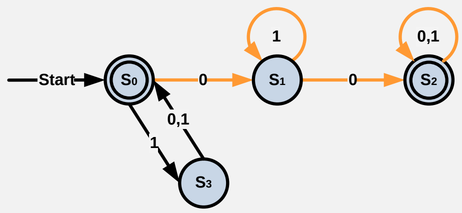
Question: Find the language recognized by the given deterministic finite-state automaton. [6 pts]



Answer: Since state S_0 is final, the empty string is accepted. Then let's do it little by little.



through this path we have $\{10 \mid 11\}^*$

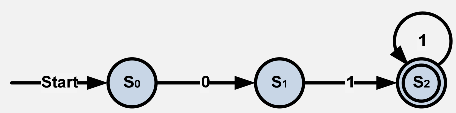


through this path we have $\{0\}\{1\}^*\{0\}\{0,1\}^*$

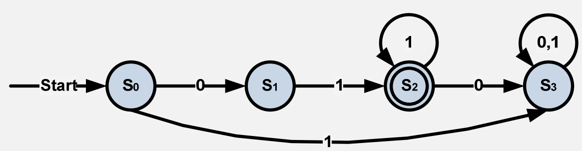
Therefore sum them together, we have $\{\lambda\} \cup \{10 \mid 11\}^* \cup \{0\}\{1\}^*\{0\}\{0,1\}^*$. The first term apparently can be omitted because it's covered in the first term. So the final solutions is $\{10 \mid 11\}^* \cup \{0\}\{1\}^*\{0\}\{0,1\}^*$.

Question: Construct a deterministic finite-state automaton that recognizes the set of all bit strings that starts with a 0, and followed by at least one 1. [Chapter 12.3 Review]

Answer: firstly, we would draw out the paths that the language defines. All accepted sentences are included in the automaton below:



Then we would take the unaccepted paths into consideration. All alphabet characters need to be covered, so we introduce paths that lead to unaccepted state.



As we can see, the state S_3 does not have any path to go to the final accepting states/state, which is S_2 in this case. So, the paths go into S_3 "dead ends".