TA: Jade Cheng

ICS 312

Homework Solution #1

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Exercise 1.2

Question: Convert to binary: 2483_d, $3E8A_h$

Answer:

2483/2	= 1241 +	1		
1241/2	= 620 +	1		
620/2	= 310 +	0		
310/2	= 155 +	0		
155/2	= 77 +	1		
77/2	= 38 +	1		
38/2	= 19 +	0	$\Rightarrow$	$2483_d = 1001\ 1011\ 0011_b$
19/2	= 9 +	1		
9/2	= 4 +	1		
4/2	= 2 +	0		
2/2	= 1 +	0		
1/2	= 0 +	1		

$3E8A_h = 0011\ 1110\ 1000\ 1010_b$

Question: Convert to decimal: $10\ 1100\ 0111_b$, $3E8A_h$

Answer:

$$0010\ 1100\ 0111_b = 1 \cdot 2^9 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 512 + 128 + 64 + 4 + 2 + 1$$

$$= 711_d$$

$$3E8A_h = 3 \cdot 16^3 + 14 \cdot 16^2 + 8 \cdot 16^1 + 10 \cdot 16^0$$

$$= 12288 + 3584 + 128 + 10$$

$$= 16010_d$$

Question: Perform the following additions in binary:

Answer:

$$
\begin{array}{r}
1101\ 0101_b \\
+\ 111\ 0011_b \\
\hline
1\ 0100\ 1000_b
\end{array}
\qquad
\begin{array}{r}
1001\ 1111_b \\
+\ 10\ 0001_b \\
\hline
1100\ 0000_b
\end{array}
$$

Question: Perform the following additions in hex:

Answer:

$$
\begin{array}{r}
9B34_h \\
+5AE6_h \\
\hline
F61A_h
\end{array}
$$

Exercise 1.3

Question: Convert the following 8-bit binary numbers to decimal

Answer: Assuming that they are signed 2's complement numbers:

$$1110\ 1011_b = -(2's\ complement\ of\ 0001\ 0101_b) = -21$$

$$1000\ 0000_b = -(2's\ complement\ of\ 1000\ 0000_b) = -128$$

$$0100\ 0101_b = 69$$

Assuming that the numbers are unsigned:

$$1110\ 1011_b = 2^7 + 2^6 + 2^5 + 2^3 + 2^1 + 2^0 = 235$$

$$1000\ 0000_b = 2^7 = 128$$

$$0100\ 0101_b = 2^6 + 2^2 + 2^0 = 69$$

Question: Convert the following 8-bit hex numbers to decimal

Answer: Assuming that they are signed 2's complement numbers:

$$FF_h = -(2's\ complement\ of\ 01_h) = -1$$

$$F0_h = -(2's\ complement\ of\ 10_h) = -16$$

Assuming that the numbers are unsigned:

$$FF_h = 15 \cdot 16^1 + 15 \cdot 16^0 = 255$$

$$F0_h = 15 \cdot 16^1 + 0 \cdot 16^0 = 240$$

Question: Do the following addition exercises by translating the numbers into 8-bit 2's complement binary numbers, performing the arithmetic, and translating the result back into a decimal number. Indicate where overflow occurs and why, based on the binary arithmetic.

Answer:

47: $0010\ 1111_b$
38: $0010\ 0110_b$
———————
$0101\ 0101_b$
⇓
$48 + 38 = 85$

47: $0010\ 1111_b$
-38: $1101\ 1010_b$
———————
$1\ 0000\ 1001_b$
⇓
$47 + (-38) = 9$

-47: $1101\ 0001_b$
-38: $1101\ 1010_b$
———————
$1\ 1010\ 1011_b$
⇓
$-(2's\ complement\ of\ 1010\ 1011_b)$
$= -(0101\ 0101_b)$
$= -85$

47: $0010\ 1111_b$
88: $0101\ 1000_b$
———————
$1000\ 0111_b$
⇓
$-(2's\ complement\ of\ 1000\ 0111_b)$
$= -(0111\ 1001_b)$
$= -120$ ***Overflow***

-47: $1101\ 0001_b$
88: $0101\ 1000_b$
———————
$0101\ 1000_b$
⇓
$-47 + 88 = 41$

47: $0010\ 1111_b$
-88: $1010\ 1000_b$
———————
$1101\ 0111_b$
⇓
$-(2's\ complement\ of\ 1101\ 0111_b)$
$= -(0010\ 1001_b)$
$= -41$

Question: Give the 16-bit 2's complement form of the following 8-bit 2's complement numbers.

Answer: $94_h = FF94_h$ because $9 > 8$, this is a negative number, the leading bits need to be F's.
$FF_h = FFFF_h$ because $F > 8$, this is a negative number, the leading bits need to be F's.

Question: Which of the following 16-bit 2's complement numbers can be shortened to 8-bit and maintain their value?

Answer: $FF94_h = 94_h$ as we've shown above.
$FF3C_h \neq 3C_h$ because $FF3C_h$ is a negative number while $3C_h$ is a positive number.

Question: A positive binary number is even precisely when its last digit is 0. When is a negative 2's complement number even?

Answer: A negative 2's complement number is even precisely when its last digit is 0 as well.

Proof part 1: Assume there exists a 2's complement negative even number which ends with 1. We can, therefore, express this number as

$$\begin{aligned} 1\cdots1 &= -(2's\ complement\ of\ 1\cdots1) \\ &= -(0\cdots1) \\ &\neq even \end{aligned}$$

Since we know precisely that a positive binary number is not even when it ends with a 1. This is a conflict with our assumption. Our assumption is, therefore, wrong.

Proof part 2: Assume there exists a 2's complement negative odd number which ends with 0. We can, therefore, express this number as

$$\begin{aligned} 1\cdots0 &= -(2's\ complement\ of\ 1\cdots0) \\ &= -(either\ 0\cdots0\ or\ 1\cdots0) \\ &= even \end{aligned}$$

Since we know precisely that a positive binary number is even when it ends with a 0. This is a conflict with our assumption. Our assumption is, therefore, wrong.

Question: There is nothing particularly magical about 2's complement in binary, other than that the computations are particularly easy in base 2. We could also do 10's complement arithmetic in normal decimal numbers. We can represent negative numbers as the 10's complement of the corresponding positive number. Do the arithmetic in problem 3 in 3 digit 10's complement decimal arithmetic.

Answer:

47:	$047_{10's}$	47:	$047_{10's}$
38:	$038_{10's}$	−38:	$962_{10's}$
	$085_{10's}$		$1\,009_{10's}$
	⇓		⇓
	$48 + 38 = 85$		$47 + (-38) = 9$

−47: $953_{10's}$
−38: $962_{10's}$

$1\,915_{10's}$

$\Downarrow$

$-(10's\ complement\ of\ 915_{10's})$
$= -(085_{10's})$
$= -85$

47: $047_{10's}$
88: $088_{10's}$

$135_{10's}$

$\Downarrow$

$47 + 88 = 135$

−47: $953_{10's}$
88: $088_{10's}$

$1\,041_{10's}$

$\Downarrow$

$-47 + 88 = 41$

47: $047_{10's}$
-88: $912_{10's}$

$959_{10's}$

$\Downarrow$

$-(10's\ complement\ of\ 959_{10's})$
$= -(041_{10's})$
$= -41$