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**ICS 312**  
**Optional Homework Solution**  
**Assigned on September 15, 2009**

**Optional Homework**

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**Question:** Find the great common divisor ( $gcd$ ) of integer 117 and 93, and express it in terms of 117 and 93,  $gcd(117, 93) = a \cdot 117 + b \cdot 93$ .

**Answer:** Apply the Extended Euclidean Algorithm to find out  $gcd(117, 93)$ :

$$117 = 1 \cdot 93 + 24$$

$$93 = 3 \cdot 24 + 21$$

$$24 = 1 \cdot 21 + 3$$

$$21 = 7 \cdot 3.$$

Apply Back Substitution we have:

$$\begin{aligned} gcd(117, 93) &= 3 \\ &= 24 - 1 \cdot 21 \\ &= 24 - 1 \cdot (93 - 3 \cdot 24) \\ &= -1 \cdot 93 + 4 \cdot 24 \\ &= -1 \cdot 93 + 4 \cdot (117 - 1 \cdot 93) \\ &= 4 \cdot 117 - 5 \cdot 93. \end{aligned}$$

**Another E.g.:** Find the great common divisor ( $gcd$ ) of integer 81 and 250, and express it in terms of 81 and 250,  $gcd(81, 250) = a \cdot 81 + b \cdot 250$ .

**Answer:** Apply the Extended Euclidean Algorithm to find out  $gcd(81, 250)$ :

$$250 = 3 \cdot 81 + 7$$

$$81 = 11 \cdot 7 + 4$$

$$7 = 1 \cdot 4 + 3$$

$$4 = 1 \cdot 3 + 1$$

$$3 = 3 \cdot 1.$$

Apply Back Substitution we have:

$$\begin{aligned} \gcd(81, 250) &= 1 \\ &= 4 - 1 \cdot 3 \\ &= 4 - 1 \cdot (7 - 1 \cdot 4) \\ &= -1 \cdot 7 + 2 \cdot 4 \\ &= -1 \cdot 7 + 2 \cdot (81 - 11 \cdot 7) \\ &= 2 \cdot 81 - 23 \cdot 7 \\ &= 2 \cdot 81 - 23 \cdot (250 - 3 \cdot 81) \\ &= -23 \cdot 250 + 71 \cdot 81. \end{aligned}$$