Planning of Manufacturing and Distribution in a Global Corporation

Jade Cheng

September 2008

University of Hawaiʻi at Mānoa
Department of Information and Computer Sciences
2500 Campus Road
Honolulu, HI 96822

Planning of Manufacturing and Distribution in a Global Corporation

Jade Cheng

Department of Information and Computer Sciences
University of Hawaiʻi at Mānoa
Honolulu, HI 96822

yucheng@hawaii.edu

September 23, 2008

Abstract

This paper is concerned with the planning of manufacturing and distribution in a global corporation. We present a numeric model designed to minimize the manufacturing cost price of a corporation's final assembly. We consider both internal and external suppliers of the corporation. We simplify and approximate ancillary parameters, while focusing on the dynamic relationships regarding facility layout and location planning. The key components are expressed as a bipartite graph consisting of a sub-graph representing the internal relationships of the facilities and another sub-graph representing the internal relationships of the locations. The model is designed to facilitate optimization of the combined entities. Our conclusions are general enough to serve as the future reference for any real-world implementation. We do not, however, provide an algorithm, implementation, or test plan with the model.

1 Introduction

This paper is concerned with the planning of manufacturing and distribution in a global corporation. This is a common investigation, not specific to any particular company, but many companies have to manage this issue. Larger companies are more likely to work globally, and larger companies have more to gain (or lose) from this sort of in-depth analysis.

We make the assumption that we are systems analysts of a consulting firm whose clients are global corporations that need to decrease costs in manufacturing of their products. It used to be clear that companies located in China or India can produce cheaper products, but labor costs have been balanced out in recent years considering the cost of shipping and oil prices. This is an interesting time in history to look into this kind of research. Although we assume we are working with just one company, the ideas presented here are general enough to be applied in many different circumstances.

Hereon we present a mathematical description of the challenge presented above as a *problem formulation*. Although our "customer" is a "global corporation," we make the assumption that a Decision Researcher with a strong mathematical background is the target reader.

This paper will not, however, provide an implementation or algorithm along with the model and problem formulation. This paper will not provide a test framework, which would be necessary to ensure the model is accurate in more than just theory. Instead, this paper will provide simplified examples, figures, and straightforward mathematical equations that appeal to one's common sense or intuition.

2 Preliminaries

Manufacturing and distribution in a global corporation is a complex system, and our goal is to represent it as a numerical model. A long-term objective would be to build a testable algorithm that could be applied to existing data, which could validate the model. As we have discussed, however, this is beyond the scope of this paper, and our short-term goal is to simply define the model.

Before we discuss the problem formulation, we will discuss key concepts, simplifications, and approximations we made to simplify and generalize this system.

2.1 Key Concepts of the Model

We are going to provide a model regarding location planning and facility layout, while simplifying or approximating other issues we feel are less important. The goal of this model is to maximize profit, and we do that by minimizing the cost per unit and maximizing the production per funding input.

For example, transportation costs favor large, infrequent shipments, while inventory costs favor small, frequent shipments. We will consider only the cheapest possible transportation between certain locations, while ignoring the dynamic relationship between transportation costs with other factors involved. We can conceptualize this with the following train of thought: If we were to keep a reasonable shipping pace, the shipping speed would not affect the manufacturing throughput; if we were to have a large enough storage capacity, bulk shipments would always be possible.

2.2 Approximations and Simplifications

Our model will simplify a number of parameters such as inventory control, customer traffic, and distribution systems. We will make the case, however, that we can approximate them adequately using our model.

For example, in the real world, discrete events occur over time. The events do not happen on a rigid schedule. Real people have good days and bad days, and this certainly affects manufacturing throughput. But we can approximate the output of a given entity by averaging the output over time. We can consider the output of an entity as a flow instead of discrete objects occurring at variable intervals.

Regarding the manufacturing and distribution within a global corporation, this is a reasonable approximation. After all, the model presented in this paper is not meant to be applied in real-time. If events arise that significantly affect the average output of certain entities, the model still works; its algorithm, or implementation, needs only to be reevaluated.

In a similar light, many real-world problems involve uncertainty, and mathematics has been extremely useful in identifying ways to manage it. Modeling uncertainty is important in risk analysis for complex systems, such as space shuttle flights, large dam operations, or nuclear power generation.[1] By representing our system as a flow model, however, this problem of random variability is greatly reduced. By not supporting a real-time solution, which is necessary in some systems but not ours, this concern becomes negligible.

Performance analysis, which is usually another concern, will not be considered in our model. Our model will not take into consideration performance metrics explicitly, but as we will see, they are implicitly represented as weighted graphs. We will assume performance metrics are in place to measure the capabilities of various entities, and the results of those metrics are used to define the throughput.

As we will demonstrate, our model relies on a heuristic value for distribution of material and products from various manufacturing plants. In the real world, the cost of transportation depends on many things such as modes of shipping (e.g., train, boat, and airplane), frequency, and quantities. These are not taken into consideration in our model, but in a real-world situation, they could not be ignored.

3 Problem Formulation

Our problem formulation is an attempt to identify our customer's problematic state, namely the desire to maximize profitability without concrete methods to achieve this goal, and to provide them a mathematical model that serves as a suitable reference for future implementations.

A working example will help formulate our model, and we begin our discussion here.

3.1 Real-life Example of Organizational Configuration

We consider a large corporation $\mathbb{C}$ that produces a product consisting of several major parts. The parts are produced at various manufacturing facilities, and each part may consist of sub-parts that

[1] http://www.maa.org/features/012304mathinindustry.html

are produced at additional manufacturing facilities. This configuration may repeat itself until a facility produces an **atomic** part, that is, a part that cannot be further divided into sub-parts. An atomic part can be purchased at other facilities, or it can be produced by a facilities already controlled by the corporation $\mathbb{C}$. **Composite** parts, parts that consist of two or more sub-parts, may also originate from a facility controlled by the corporation $\mathbb{C}$, or they may be purchased from external sources.

We consider three costs associated with facilities. First is the **operational fee**. Use of a facility requires a certain minimal cost, regardless of the amount of parts produced at that facility. This might also consist of the cost required to build the facility. This cost applies only to facilities under the control of corporation $\mathbb{C}$.

The second cost to consider is the **production cost**. This depends on the type of part produced and the origin and cost of the sub-parts used to produce it.

The third cost is the **transportation cost** to move the parts from location to location. This depends mainly on the geographic locations of the facilities.

Therefore, we consider there are three types of facilities from which the corporation could choose to obtain parts. The first kind is a factory that is controlled by the corporation. They produce atomic parts and supply the final assembly factory. These kinds of factories directly contribute to the final product. We will call them **Internal Atomic Suppliers**. Internal Atomic Suppliers would cost the corporation all three kinds of fees, operational fees, production costs, and transportation costs.

The second kind of facility is an external company that provides services or sub-parts that contribute to the final product. The corporation makes direct purchases from them without worrying about the factory operational fees. We will call them **External Suppliers**. External Suppliers would cost the corporation two different kind of fees, production costs and transportation costs.

The third kind of facility is a factory that is controlled by the corporation. The difference between this kind of facility and the Internal Atomic Supplier is that their products are composite parts. Therefore they have sub-suppliers. The sub-suppliers could be controlled by either the company or external sources. We will them **Internal Composite Supplier**s. Internal Composite Suppliers not only cost all three kinds of fees, their sub-suppliers would generate cost, too.

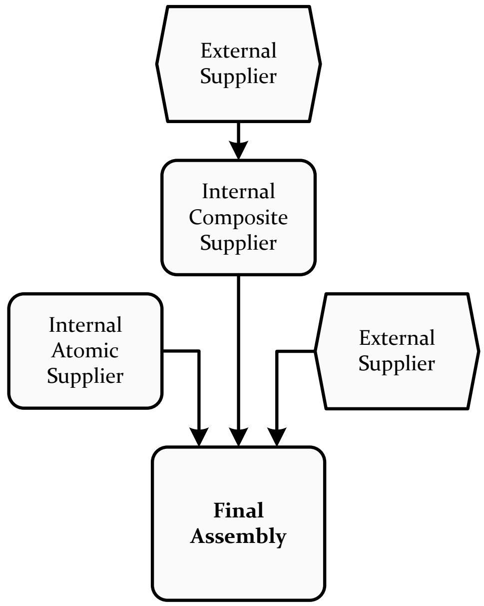

FIGURE 1: Organization of Parts and Manufacturing Facilities.

Obviously, the relationship of facilities could be a bit more complicated than what is shown in Figure 1. Namely, the suppliers might have communication with other suppliers or sub-suppliers. The sub-suppliers could also be considered as the first-level suppliers at the same time.

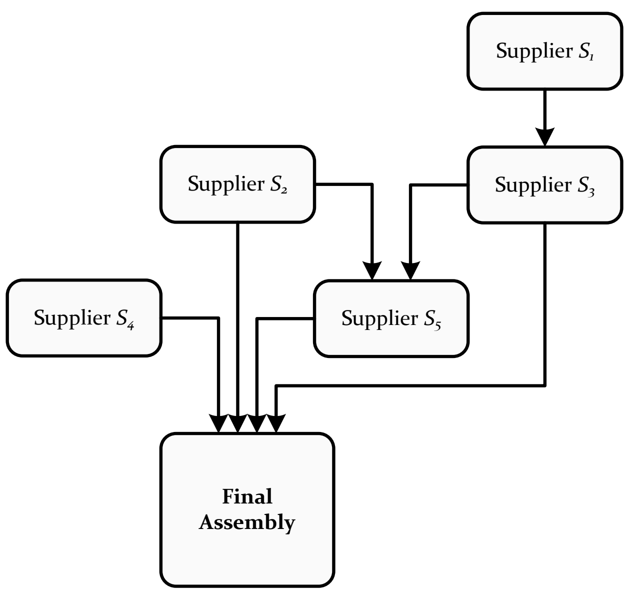

FIGURE 2: More Complicated Organization of Parts and Manufacturing Facilities.

For example, in Figure 2, let us suppose supplier S_2 produces Lithium batteries. Their product is needed for supplier S_5, which produces microcontrollers. The Final Assembly, which produces fancy remote controls for high-end television sets, requires both the batteries and the microcontrollers, so S_2 supplies two nodes in the same organizational tree.

3.2 Configuration Setup for the Model

In our model, we do not have to consider the levels of all different kinds of facilities. What we care about is the fact that each of them has to finish a task, and each of them has to take a location which determines the transportation cost between that certain facility and the rest of the world.

The corporation has a number of tasks to accomplish. It needs to acquire the sub-parts of the final product. Some of the tasks might not have any direct relationship with the final product, but we don't have to specify what exactly the relationship is. We could just as well assign this task a relationship to the final product as "N/A". Actually, we can assign relationships of this task to all other tasks, including the task of the final assembly corporation, which is in charge of putting the final product together.

The corporation also has a number of locations to consider. As a global corporation, they have facilities in more than one country. The cost varies from location to location to transport goods. Some locations might be very close, perhaps even two departments of the same factory that share the same storage space. Regardless, as long as they are assigned with two different tasks, we consider them as two different locations. In this case, we would simply assign a transportation cost of zero to that relationship.

The corporation does not get to choose the location of external suppliers. In other words, the corporation does not get to minimize the transportation cost once they decide to use a certain manufacturer. But they can choose the most cost-effective ones after considering both product prices and transportation costs. Therefore, we assign a transportation cost relationship between each available manufacturer and the factory that is controlled by the corporation.

3.3 Process and Reasoning

We have two cases that are fairly independent to one another. The descriptions of these two parts of our model are described here.

3.3.1 Case 1 — Facilities Controlled by the Corporation

To model this case, we consider the corporation has a collection of tasks to accomplish and a collection of possible factory locations to choose from. Performing a certain task at a certain location requires a certain amount of cost, which would most likely change for different pairings. We must also consider individual factory operational fees at varying locations, and we must consider the transportation cost from one location to another.

And lastly, we must consider what we will call the transportation flow size between two tasks. We define the **transportation flow size** as the amount of product to move from location to location per unit of time. It is a type of manufacturing throughput.

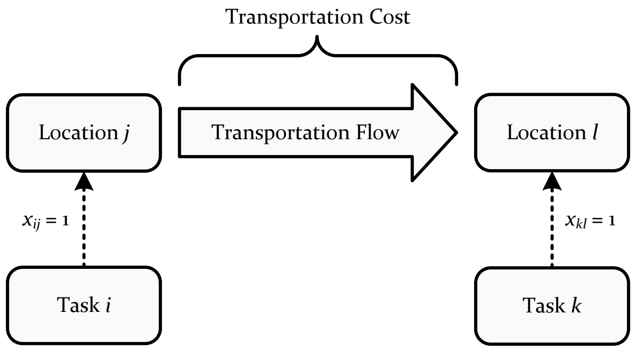

FIGURE 3: Assignment of Tasks to Internal Locations.

With this in mind, we can begin to define the inputs and outputs of this system more formally.

Input

There are n different locations to choose for building factories, and there are n tasks that need to be done in specialized factories. We consider an n element set $\mathbb{A} = \{1, 2, \dots, i, \dots, k, \dots, n\}$. This represents each individual task. And we consider another n element set $\mathbb{B} = \{1, 2, \dots, j, \dots, l, \dots, n\}$. This represents each individual location.

$T = (t_{ik})$, where t_{ik} is the transportation size from task i and task j. The unit is Q pieces.

$D = (d_{jl})$, where d_{jl} is the transportation cost from location j to location l. The unit is K dollars per Q pieces.

$P = (p_{ij})$, where $p_{ij}$is the cost price for task i to be done at location j. The unit is K dollars per Q pieces.

$C = (c_{ij})$, where c_{ij} is the factory construction fee for task i and location j. The unit is K dollars.

We define an indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

Output

This first case can be expressed as the minimum of the formula below.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} t_{ik} \cdot (d_{jl} + p_{ij}) \cdot x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \quad (1)$$

3.3.2 Case 2 — Facilities Not Controlled by the Corporation

To model this case, we consider the corporation has a collection of external tasks to accomplish and a collection of available manufacturers to choose from. Performing a certain task at a certain external supplier requires a certain amount of cost. We must also consider the transportation cost from certain external suppliers to its destination. And as it was with case 1, we must also consider the transportation flow size.

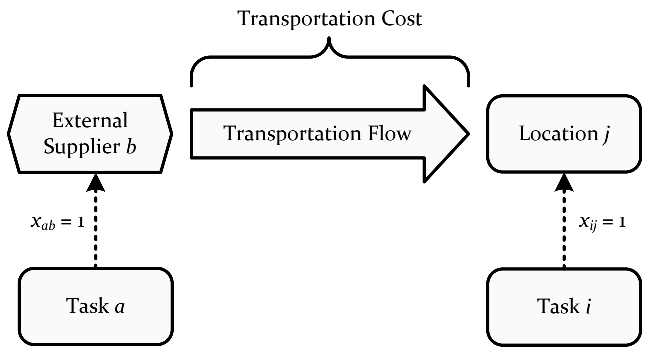

FIGURE 4: Assignment of Tasks to External Suppliers.

Again, we can now define the inputs and outputs of this case more formally.

Input

We consider an m element set $\mathbb{C} = \{1, 2, \ldots, a, \ldots, m\}$. This represents the tasks need to be done by an already-existing manufacturer. We consider an o element set $\mathbb{D} = \{1, 2, \ldots, b, \ldots, o\}$. This represents the manufactures that are capable of performing any of these tasks.

$T' = (t'_{ai})$, where t'_{ai} is the transportation size from manufacturer task a to task i. The unit is Q pieces.

$D' = \left(d'_{bj}\right)$, where d'_{bj} is the transportation cost from manufacture b to location j. The unit is K dollar per Q pieces.

$P' = (p'_{ab})$, where p'_{ab} is the cost price for task a to be done at manufacturer b. The unit is K dollar per Q pieces.

We define an indicator random variable $x'_{ab} = \begin{cases} 1 \\ 0 \end{cases}$ such that if manufacture a is assigned to task b, then $x'_{ab} = 1$; if manufacture a is not assigned to task i, then $x'_{ab} = 0$.

We still have the indicator random variable $x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ such that if task i is assigned to location j, then $x_{ij} = 1$; if task i is not assigned to location j, then $x_{ij} = 0$.

Output

This second case can be expressed as the minimum of the formula below.

$$\mathbb{O} = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{b=1}^{o} t'_{ai} \cdot \left(d'_{aj} + p'_{ab}\right) \cdot x'_{ab} x_{ij} \tag{2}$$

3.3.3 Combined Cases

Now that we have formal definitions for both case 1 and case 2, we can combine these definitions to produce a formula that describes the system as a whole. Although the formula is long, it is obtained in a straightforward manner; we simply sum the two equations to arrive at the formula below.

$$\mathbb{O} = \min \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} t_{ik} \cdot \left(d_{jl} + p_{ij}\right) \cdot x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{b=1}^{o} t'_{ai} \cdot \left(d'_{aj} + p'_{ab}\right) \cdot x'_{ab} x_{ij} \right] \tag{3}$$

3.4 A Simple Example of the Model

Let us assume we have three sub-parts needing to be built and that we have three locations at which those sub-parts can be assembled. Before we can know the optimal assignment, we must collect some heuristic values and apply them to our system.

3.4.1 The Internal Relationships of Three Tasks

Each task manager must decide whether or not the finished product of that task should be transported to another factory. We can represent this using the following table, which shows the transportation flow size moving from tasks i to k. Units are in Q pieces.

t_{ik}	***A***	***B***	***C***
A	—	10	7
B	0	—	13
C	20	0	—

3.4.2 The Internal Relationships of Three Locations

Each location manager must measure the transportation cost to all other locations. We can represent this using the following table, which shows the transportation cost from location j to l. Units are in K dollars per Q pieces.

d_{jl}	***a***	***b***	***c***
a	—	20	30
b	25	—	40
c	15	35	—

3.4.3 The Cost Price for Locations to Finish Tasks

Each location manager and each task manager must estimate the cost price of performing their particular task at their particular location. We can represent this relationship between task i and location j using the following table. Units are in K dollars per Q pieces.

p_{ij}	***A***	***B***	***C***
a	5	10	20
b	6	8	28
c	7	5	22

3.4.4 The Operational Fees at Different Locations

Each location manager and each task manager must also estimate the factory operational fee of performing their particular task at their particular location. We can represent this relationship between task i and location j using the following table. Units are in K dollars.

c_{ij}	***A***	***B***	***C***
a	200	700	1000
b	300	650	1200
c	400	600	900

3.4.5 Visualization of the Model

At this point, we can visualize the internal relationships of tasks and the internal relationships of locations as two separate weighted graphs.

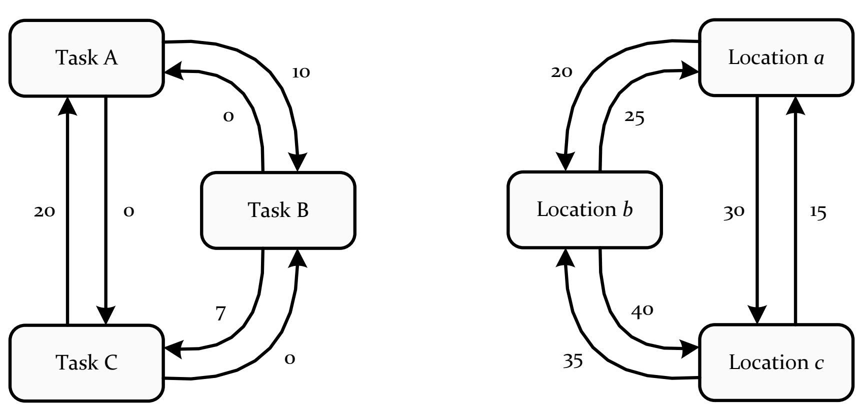

FIGURE 5: Bipartite Graph Representing Tasks and Locations.

3.4.6 Plans of Action

Clearly, we have six different combinations of how to assign tasks to locations. They are represented in the following table.

Plan 1	$A \to a$	$B \to b$	$C \to c$
Plan 2	$A \to a$	$B \to c$	$C \to b$
Plan 3	$A \to b$	$B \to a$	$C \to c$
Plan 4	$A \to b$	$B \to c$	$C \to a$
Plan 5	$A \to c$	$B \to a$	$C \to b$
Plan 6	$A \to c$	$B \to b$	$C \to a$

We shall compare two of these plans. Arbitrarily, we will choose Plan 1 and Plan 4.

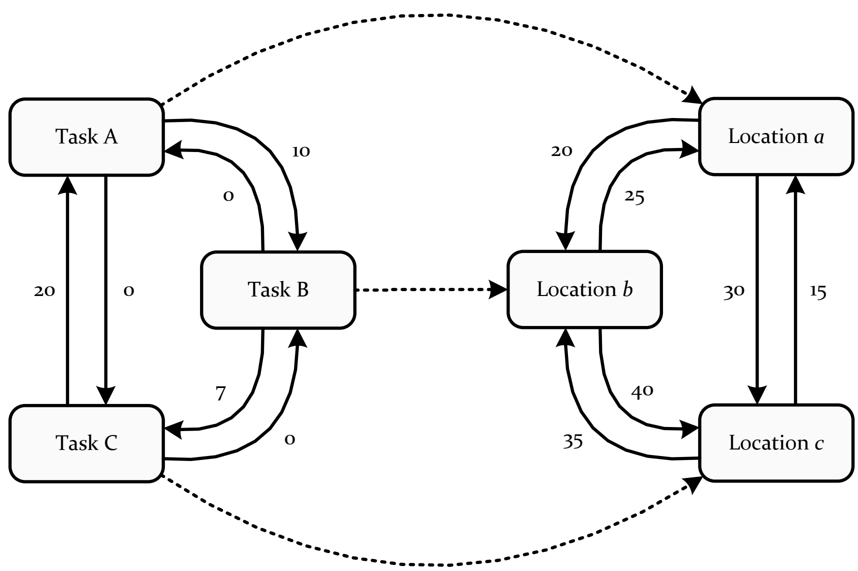

FIGURE 6: Plan 1 Shown as a Weighted Bipartite Graph.

In Plan 1, Task A is assigned to location a, Task B is assigned to Location b, and Task C is assigned to Location c. Applying equation (1), we get

$$\mathbb{O} = t_{AB} \cdot (d_{ab} + p_{aA}) + t_{AC} \cdot (d_{ac} + p_{aA}) + t_{BC} \cdot (d_{bc} + p_{bB}) + t_{BA} \cdot (d_{ba} + p_{bB}) + t_{CA} \cdot (d_{ca} + p_{cC}) + t_{CB} \cdot (d_{cb} + p_{cC}) + c_{aA} + c_{bB} + c_{cC}$$

$$= 10 \times (5 + 20) + 7 \times (5 + 30) + 13 \times (8 + 40) + 20 \times (22 + 15) + 200 + 650 + 900$$

$$= 3609$$

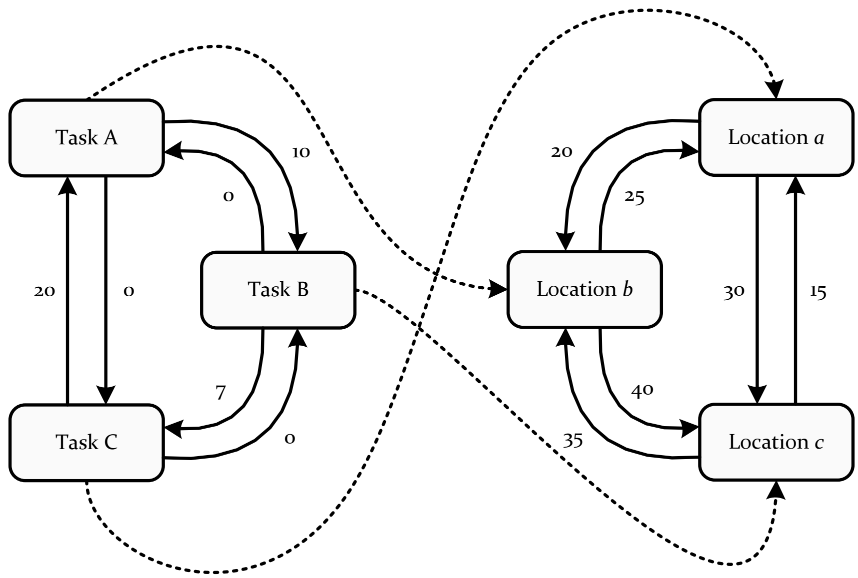

FIGURE 7: Plan 4 Shown as a Weighted Bipartite Graph.

In Plan 4, Task A is assigned to location b, Task B is assigned to Location c, and Task C is assigned to Location a. Applying equation (1), we get

$$\mathbb{O} = t_{AB} \cdot (d_{bc} + p_{bA}) + t_{AC} \cdot (d_{ba} + p_{bA}) + t_{BC} \cdot (d_{ca} + p_{cB}) + t_{BA} \cdot (d_{cb} + p_{cB}) + t_{CA} \cdot (d_{ab} + p_{cA}) + t_{CB} \cdot (d_{ac} + p_{cA}) + c_{bA} + c_{cB} + c_{cA}$$

$$= 10 \times (6 + 40) + 7 \times (6 + 25) + 13 \times (5 + 15) + 20 \times (20 + 20) + 300 + 600 + 1000$$

$$= 3637$$

Therefore, between Plan 1 and Plan 4, Plan 1 is more cost-effective than plan. To find the optimal solution, however, we would need to evaluate all plans and take the minimum value obtained.

4 Concluding Remarks

In this paper, an optimal solution model is built to serve as a future implementation reference for our global corporation client in order to achieve the most cost-effective manufacturing and distribution business planning.

The model is constructed under a series of preliminaries. We considered most of the possible factors affecting optimization in models like our own, but we analyzed only the key conceptions and limitations in depth.

The problem is formulated as a weighted bipartite graph representing facility layout and location planning for both the internal and external suppliers of the corporation. The formulas allow for optimization by minimizing sums based on the weights of the graph.

Neither an algorithm nor an implementation was provided, and as such, the model is not meant to be used directly in real life. Instead it is meant to serve as a simplified model of a real-world problem.