Student: Yu Cheng (Jade)

Math 311

Homework #4

September 23, 2009

Section 3.2

Question: W is a subset of R^2 consisting of vectors of the form

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

In each case determine whether W is a subspace of R^2. If W is a subspace, then give a geometric description of W.

2. $W = \{x: x_1 - x_2 = 2\}$

Answer: _Question 1:_ Is the zero vector in W? No because $0 - 0 \neq 2$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin W, \theta \notin W$.

Conclusion: W is not a subspace.

8. $W = \{\boldsymbol{x}: x_1 x_2 = 0\}$

Answer: _Question 1:_ Is the zero vector in W? Yes because $0 \cdot 0 = 0$, so $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W, \theta \in W$.

Question 2: If the vector $x \in W$ and the vector $y \in W$, is $(x + y) \in W$? No, an counter example can be, $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ while $x + y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W$.

Conclusion: W is not a subspace.

Question 18: Let a be a fixed vector in R^3, and define W to be the subset of R^3 given by

$$W = \{x: a^T x = 0\}$$

Prove that W is a subspace of R^3.

Answer: _Question 1:_ Is the zero vector in W? Yes because $a^T \theta = 0$, so, $\theta \in W$.

Question 2: If the vector $x \in W$ and the vector $y \in W$, is $(x + y) \in W$?

$$x \in W \Rightarrow a^T x = 0$$

$$y \in W \Rightarrow a^T y = 0$$

$$\Rightarrow a^T x + a^T y = 0$$

$$\Rightarrow a^T (x + y) = 0 \text{ (according to Theorem } 9 - d)$$

$$\Rightarrow x + y \in W\,.$$

Question 3: If the vector $x \in W$ is $kx \in W$ where $k \in \mathbb{R}$?

$$x \in W \Rightarrow a^T x = 0$$

$$\Rightarrow k \cdot a^T x = 0$$

$$\Rightarrow a^T (kx) = 0 \text{ (accoring to Theorem } 9 - c)$$

$$\Rightarrow kx \in W\,.$$

Conclusion: W is a subspace.

Question 30: If U and V are subsets of R^n, then the set $U + V$ is defined by

$$U + V = \{x : x = u + v, u \text{ in } U, \text{and } v \text{ in } V\}$$

Prove that if U and V are subspaces of R^n, then $U + V$ is also a subspace of R^n.

Answer: *Question 1:* Is the zero vector in $U + V$? Yes because $\theta \in U, \theta \in V \Rightarrow (\theta + \theta) \in (U + V) \Rightarrow \theta \in (U + V)$.

Question 2: If the vector $x \in (U + V)$ and the vector $y \in (U + V)$, is $(x + y) \in (U + V)$?

$$x \in (U + V) \Rightarrow x = u_x + v_x$$

$$y \in (U + V) \Rightarrow x = u_y + v_y$$

$$\Rightarrow x + y = (u_x + v_x) + \left(u_y + v_y\right)$$

$$= \left(u_x + u_y\right) + \left(v_x + v_y\right)$$

$$= u_{xy} + v_{xy} \text{ (since both } U \text{ and } V \text{ are subspaces)}\,.$$

$$\because \left(u_{xy} + v_{xy}\right) \in (U + V)$$

$$\therefore (x + y) \in (U + V)\,.$$

Question 3: If the vector $x \in (U+V)$ is $kx \in (U+V)$ where $k \in \mathbb{R}$?

$$x \in (U+V) \Rightarrow x = u+v$$

$$\Rightarrow k \cdot x = k \cdot (u+v)$$

$$= ku + kv$$

$$= u' + v' \text{ (since both } U \text{ and } V \text{ are subspaces) .}$$

$$\because (u'+v') \in (U+V)$$

$$\therefore kx \in (U+V) \text{ .}$$

Conclusion: If U and V are subspaces of R^n, then $U+V$ is also a subspace of R^n.

Question 31: Let U and V be subsets of R^n. Prove that the intersection, $U \cap V$ is also a subspace of R^n.

Answer: *Question 1:* Is the zero vector in $U \cap V$? Yes because $\theta \in U, \theta \in V \Rightarrow \theta \in U \cap V$.

Question 2: If the vector $x \in U \cap V$ and the vector $y \in U \cap V$, is $(x+y) \in U \cap V$?

$$x \in U \cap V \Rightarrow x \in U, x \in V$$

$$y \in U \cap V \Rightarrow y \in U, y \in V$$

$$\Rightarrow (x+y) \in U \text{ (since } x \in U, y \in U)$$

$$\Rightarrow (x+y) \in V \text{ (since } x \in V, y \in V)$$

$$\Rightarrow (x+y) \in U \cap V \text{ .}$$

Question 3: If the vector $x \in U \cap V$ is $kx \in U \cap V$ where $k \in \mathbb{R}$?

$$x \in U \cap V \Rightarrow x \in U, x \in V$$

$$\Rightarrow kx \in U \text{ (since } x \in U)$$

$$\Rightarrow kx \in V \text{ (since } x \in V)$$

$$\Rightarrow kx \in U \cap V \text{ .}$$

Conclusion: If U and V be subsets of R^n. The intersection, $U \cap V$ is also a subspace of R^n.