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Math 412
Homework #1
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Problem 1: For each of the following groupoids, determine the following.

a. $G = \langle \mathbb{Z}_3, + \rangle$

Answer $\langle \mathbb{Z}_3, + \rangle$'s characteristic table is shown as below. It is *commutative*, since $a + b = b + a$ for all $a \in G, b \in G$, and the table is symmetric across its diagonal. It is *associative*, since $a + (b + c) = (a + b) + c$ for all $a \in G, b \in G$, and $c \in G$. There is an *identity element* 0, since $a + 0 = 0 + a = a$ for all $a \in G$. There is *no zero element*, since there is no such element a that $b + a = a + b = a$ for all $b \in G$. There is *one idempotent element* 0, since only $0 + 0 = 0$. For equation $ax = b$, *there is a solution* for every pair of a, b, since every row/column contains all elements in the collection.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

b. $G = \langle \mathbb{Z}_4, + \rangle$

Answer $\langle \mathbb{Z}_4, + \rangle$'s characteristic table is shown as below. It is *commutative*, since $a + b = b + a$ for all $a \in G, b \in G$, and the table is symmetric across its diagonal. It is *associative*, since $a + (b + c) = (a + b) + c$ for all $a \in G, b \in G$, and $c \in G$. There is *an identity element* 0, since $a + 0 = 0 + a = a$ for all $a \in G$. There is *no zero element*, since there is no such element a that $b + a = a + b = a$ for all $b \in G$. There is *one idempotent elements* 0, since only $0 + 0 = 0$. For equation $ax = b$, *there is a solution* for every pair of a, b, since every row/column contains all elements in the collection.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

c. $G = \langle \mathbb{Z}_3, \times \rangle$

Answer $\langle \mathbb{Z}_3, \times \rangle$'s characteristic table is shown as below. It is *commutative*, since $a \times b = b \times a$, where $a \in G, b \in G$, and the table is symmetric across its diagonal. It is *associative*, since $a \times (b \times c) = (a \times b) \times c$ for all $a \in G, b \in G$, and $c \in G$. There is *no identity element*, since no such element a that $a \times b = b \times a = b$ for all $b \in G$. There is *a zero element* 0, since $a \times 0 = 0 \times a = 0$ for all $a \in G$. There is *two idempotent elements* 0 *and* 1, since $0 \times 0 = 0$ and $1 \times 1 = 1$. For equation $ax = b$, *there isn't always a solution* for every pair of a, b, since $0x = 1, 0x = 2$ don't have any solutions.

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

d. $G = \langle \mathbb{Z}_4, \times \rangle$

Answer $\langle \mathbb{Z}_4, \times \rangle$'s characteristic table is shown as below. It is *commutative*, since $a \times b = b \times a$, where $a \in G, b \in G$, and the table is symmetric across its diagonal. It is *associative*, since $a \times (b \times c) = (a \times b) \times c$, where $a \in G, b \in G$, and $c \in G$. There is *no identity element*, since $0 \times 1 \neq 1$, while $a \times 1 = 1 \times a = a$ for all $a \in G, a \neq 0$. There is *one zero element* 0, since $a \times 0 = 0 \times a = 0$ for all $a \in G$. There is *two idempotent elements* 0 *and* 1, since only $0 \times 0 = 0, 1 \times 1 = 1$. For equation $ax = b$, *there isn't always a solution* for every pair of a, b, since $0x = 1, 0x = 2$ don't have any solutions.

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

e. The set $G = \{0, 1, 2, 3, 4\}$ with the multiplication $x \circ y = x$.

Answer The characteristic table for the given set is shown as below. It is *not commutative*, since $a \circ b \neq b \circ a$ for all $a \in G, b \in G$, and the table is not symmetric across its diagonal. It is *associative*, since $a \circ (b \circ c) = a = (a \circ b) \circ c$ for all $a \in G, b \in G$, and $c \in G$. There is *no identity element*, since there is no such element a that $a \circ b = b \circ a = b$ for all $b \in G$. There is *no zero element*, since there is no such element a that $a \circ b = b \circ a = a$ for all $b \in G$. They are all *idempotent elements*, since $a \circ a = a$ for all $a \in G$. For equation $ax = b$, *there isn't always a solution* for every pair of a, b. For example, $1 \circ x = 2$ doesn't have any solutions.

$\circ$	0	1	2	3	4
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4

f. Set G is defined as the natural numbers $\mathbb{N}$ with the operations $x \wedge y$ being the minimum of x and y.

Answer The characteristic table for the given set is shown as below. It is *commutative*, since $a \wedge b = b \wedge a = \min\{a, b\}$ for all $a \in G, b \in G$, and the table is symmetric across its diagonal. It is *associative*, since $a \wedge (b \wedge c) = \min\{a, b, c\} = (a \wedge b) \wedge c$ for all $a \in G, b \in G$, and $c \in G$. There is *an identity element, the largest element, l, in the set*, since $l \wedge b = b \wedge l = b$ for all $b \in G$. There is *a zero element 0*, $a \wedge 0 = 0 \wedge a = 0$ for all $a \in G$. They are *all idempotent elements*, since $a \wedge a = a$ for all $a \in G$. For equation $ax = b$, *there isn't always a solution* for every pair of a, b. For example, $1 \wedge x = 2$ doesn't have any solutions.

$\wedge$	0	1	2	3	$\cdots$
0	0	0	0	0	$\cdots$
1	0	1	1	1	$\cdots$
2	0	1	2	2	$\cdots$
3	0	1	2	3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	.

g. Jupiter

$\star$	R	P	S
R	R	P	R
P	P	P	S
S	R	S	S

Answer It is *commutative*, since the characteristic table is symmetric across its diagonal indicating $a \star b = b \star a$ for all $a \in Jupiter, b \in Jupiter$, and. It is *not associative*. A counter example is $R(PS) = RS = R \neq (RP)S = PS = S$. There is *no identity element*, since no such element $a \in Jupiter$, that $a \star b = b \star a = b$ for all $b \in Jupiter$. There is *no zero element*, sine no such element $a \in Jupiter$, that $a \star b = b \star a = a$ for all $b \in Jupiter$. They are *all idempotent elements*, since $a \star a = a$ for all $a \in Jupiter$. For equation $ax = b$, *there isn't always a solution* for every pair of a, b. For example, $P \star x = R$ doesn't have any solutions.

h. Saturn

$\circ$	R	P	S
R	S	P	R
P	P	R	S
S	R	S	P

Answer It is *commutative*, since the characteristic table is symmetric across its diagonal indicating $a \circ b = b \circ a$ for all $a \in Saturn, b \in Saturn$, and. It is *not associative*. A counter example is $R(PS) = RS = R \neq (RP)S = PS = S$. There is *no identity element*, since no such element $a \in Saturn$, that $a \circ b = b \circ a = b$ for all $b \in Saturn$. There is *no zero element*, sine no such element $a \in Saturn$, that $a \circ b = b \circ a = a$ for all $b \in Satrun$. They are *no idempotent element*, since no such element that $a \circ a = a$ where $a \in Saturn$. For equation $ax = b$, *there is a solution* for every pair of a, b, since every row/column contain all elements.

i. Uranus

$\diamond$	R	P	S	T
R	T	P	R	T
P	P	T	S	T
S	R	S	T	T
T	T	T	T	T

Answer It is *commutative*, since the table is symmetric across its diagonal indicating $a \diamond b = b \diamond a$ for all $a \in Uranus, b \in Uranus$, and. It is *not associative*. A counter example is $R(PS) = RS = R \neq (RP)S = PS = S$. There is *no identity element*, since no such element $a \in Uranus$, that $a \diamond b = b \diamond a = b$ for all $b \in Uranus$. There is *a zero element T*, since $T \diamond b = b \diamond T = T$ for all $b \in Uranus$. They is *one idempotent element T*, since $T \diamond T = T$. For equation $ax = b$, *there is not always a solution* for every pair of a, b, for example, $S \diamond x = P$ doesn't have any solutions.

j. Neptune

$\bullet$	R	P	S	T
R	T	P	R	R
P	P	T	S	P
S	R	S	T	S
T	R	P	S	T

Answer It is *commutative*, since the characteristic table is symmetric across its diagonal indicating $a \bullet b = b \bullet a$ for all $a \in Neptune, b \in Neptune$, and. It is *not associative*. A counter example is $R(PS) = RS = R \neq (RP)S = PS = S$. There is an *identity element T*, since $T \bullet b = b \bullet T = b$ for all $b \in Neptune$. There is *no zero element*, since there is no such element a that $a \bullet b = b \bullet a = a$ for all $b \in Neptune$. They is *one idempotent element T*, since $T \bullet T = T$. For equation $ax = b$, *there is not always a solution* for every pair of a, b, for example, $R \bullet x = S$.